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Best wishes

CHKWilliamson

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14. ABSTRACT
Of central importance to this research program are the dynamics and instabilities of streamwise vortices interacting with surfaces. Of primary interest are not only the discovery of new natural instabilities coming from vortex-vortex or vortex-surface interactions, but also ultimately the possibility to control these flows. Applications of such flows include tip vortices and junction vortices interacting with hull surfaces, their interaction with boundary layers, as well as design of vortex generators to modify surface pressures. We find a short wave instability of the secondary vortices that are created by the interaction of the primary vortices with a wall (e.g. Harris & Williamson, 2012, *J. Fluid Mechanics*). Further research concerns the influence of a wall on the long wavelength instability in a vortex pair. Three regimes are found, depending on the initial height of the vortex pair above the wall. The key to the significant reorganization of vortex structure, is the rapid circulation decay at regions along the vortex span which first come into contact with the wall, causing strong axial pressure gradients and periodic axial flows. The process of vortex reconnection causes vortex rings to rise away from the surface, in a 3D version of the "vortex rebound" in 2D vortex dynamics. Many of the discoveries of phenomena in this work are seen for the first time, and lead to a number of publications (including 2 papers for *J. Fluid Mechanics*). Also, during the period of this research, we have been invited to write a review paper on vortex pair dynamics for *Annual Review of Fluid Mechanics*, which will appear in 2016 (Leweke, LeDizes & Williamson, 2016).

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Vortex-Surface Interactions: Vortex Dynamics and Instabilities

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ABSTRACT

Of central importance to this research program is the fundamental problem of the dynamics and instabilities of streamwise vortices (mostly vortex pairs) interacting with surfaces. Of primary interest are not only the discovery of new natural instabilities coming from vortex-vortex or vortex-surface interactions, but also ultimately the possibility to control these flows. Applications of such flows include tip vortices and junction vortices interacting with hull surfaces, their interaction with boundary layers, as well as design of vortex generators to modify surface pressures. Our progress during the period of support was to discover some new and fundamental aspects of the temporal development of counter-rotating vortex pairs with a surface. The first principal result concerns a short wave instability of the secondary vortices that are created by the interaction of the primary vortices with a wall (e.g. Harris & Williamson, 2012, *J. Fluid Mechanics*). Further research concerns the influence of a wall on the long wavelength instability in a vortex pair. We find a critical height which dictates whether the vortex pair changes topologically into a system of vortex rings (the "Crow" instability), or whether this instability becomes inhibited by the wall. Three regimes are found, depending on the initial height of the vortex pair above the wall. In Regime 1 for small heights, weak axial flows are found, and the two secondary vortices interact to form large vertical rings. Regime 2 comprises a stronger axial flow, which strips away much of the primary vortices, creating concentrated vortex rings (2 per instability wavelength). In Regime 3, at higher initial heights, the primary vortices pinch off to form Crow instability vortex rings prior to wall interaction.

Intrinsic to all modes are strong periodic axial flows in the vortices, and the subsequent formation of vortex rings. Related phenomena in other different flows suggests that these are generic features of 3D vortex-wall interactions. The key to the significant reorganization of vortex structure, is the rapid circulation decay at regions along the vortex span which first come into contact with the wall. Such a region causes an axial pressure gradient, driving fluid and vorticity away from this point, and thereby periodic axial flows are formed, which dramatically reorganize the primary vorticity. The process of vortex reconnection causes vortex rings to rise away from the surface, in a three-dimensional version of what has become known as "vortex rebound" in 2D vortex dynamics. Many of the discoveries of phenomena in this work are seen for the first time, and lead to a number of publications (including 2 papers for *J. Fluid Mechanics*). Also, during the period of this research, we have been invited to write a review paper on vortex pair dynamics for *Annual Review of Fluid Mechanics*, which will appear in 2016 (Leweke, LeDizes & Williamson, 2016).

1. Technical Objectives

The Technical Objectives are an understanding of the interactions between coherent vortex configurations and walls. This research is fundamental to coherent turbulent flows in proximity to a wall. The work also has applications to streamwise vortices close to vehicle surfaces. Such vortex-surface arrangements include, for example, tip vortices or junction vortices, as well as applications to the physics of vortex generators. The interaction of perturbed longitudinal vortices with surfaces has had surprisingly little attention, despite the fact that streamwise vortices adjacent to a surface have practical and fundamental application.

This is certainly a fertile area for research, bringing into focus new and fascinating vortex dynamics and instabilities. Ultimately, further understanding of the vortex-surface interactions and overall flow field around the hull of a floating or submerged vehicle, including the distribution of forces and moments along the hull during maneuvers, is of interest to naval operations.

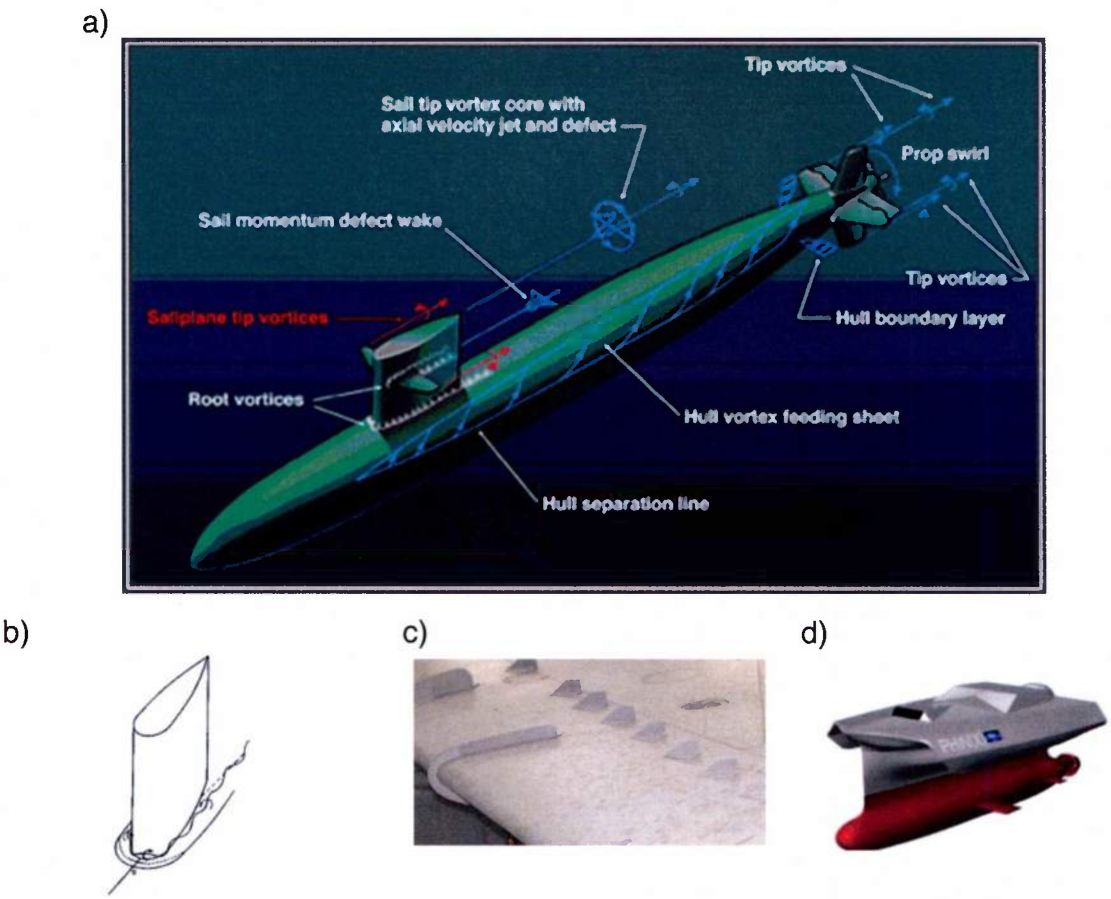


Figure 1: (a) Main vortex structures developing on a typical submarine hull; (b) Schematic illustrating a horseshoe vortex at a wing-body junction of a "Rood" airfoil (Simpson, 2001); (c) (solid) Vortex generators for separation control on the wing of a Sea Harrier VTOL jet; (d) A modern semisubmersible design (from Flagg & Joslin, 2007).

Whenever a longitudinal vortex (or vortex pair) is in proximity with a wall, secondary vorticity is produced by flow separation, to form concentrated secondary vortices of opposite sign to the primaries. A major objective is to understand the interactions between the three constituents of these flows; namely the primary vorticity, the secondary vorticity, and the wall. Interactions give rise to new and distinct three-dimensional phenomena. The physical mechanisms leading to strong periodic axial flows, and the formation of concentrated periodic vortex rings, apparently ubiquitous in these flows, have been studied in the period of support, although there is further research, from both computation and experiment which would be very useful.

The initial objectives, which we have followed in this period of research, have been to study the *temporal* development of vortex configurations interacting with a surface. In further studies, the dynamics of secondary vorticity and the development of 3D instabilities, which are so evident in the temporal flow, will be directly compared with the spatially developing flow field. This can help determine which effects are due solely to vorticity interaction, as distinct from those effects normally assumed to be due to the presence of a laminar or turbulent boundary layer. This can be done by generating trailing vortex pairs behind wing sections, including delta wings.

Ultimately, these vortex-wall interactions will lead to greater understanding of flows associated with underwater bodies with maneuvering surfaces or junctions, and flows where some control might be possible in the presence of configurations of vortex generators or other surface perturbations. One final objective is to communicate these new results, and to review the general field of research pertaining to vortex pair dynamics and instabilities in a comprehensive review, invited by *Annual Review of Fluid Mechanics* (for 2016).

2. Technical Approach

Our technical approach is principally experimental, although it is essential to include both analytical and computational studies to understand the vortex dynamics and instabilities. The experimental research uses principally our Vortex Generator Facility (see Figure 2), which comprises a pair of rotating flaps, hinged to a common base, used to generate slender horizontal vortex pairs in a 10-foot long water tank. The vortices can interact either with each other, or with parallel or oblique submerged surfaces. Principal tools in the Vortex Generator tank comprise Laser-Induced Fluorescence (LIF) and Particle Image Velocimetry (PIV) to determine velocity and vorticity fields.

Up to the present time, our research has made extensive use of the Vortex Generator tank, although in previous works (Miller & Williamson 1995), we have generated vortices successfully from a towed or self-gliding delta wing in an XY Towing Tank. Future research will employ longitudinal vortices, in wall proximity, generated by flying a delta wing close to a boundary in our computer-controlled XY Towing Tank, as well as in our XY Flowing Tank.

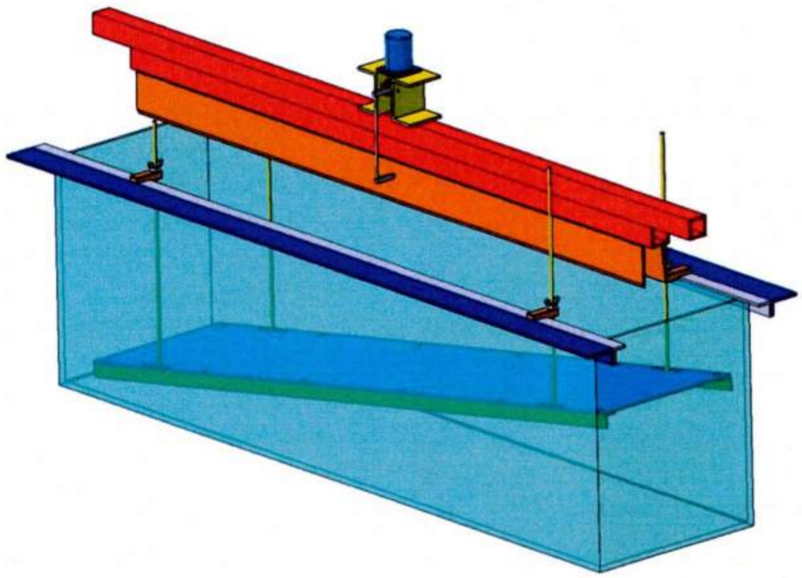
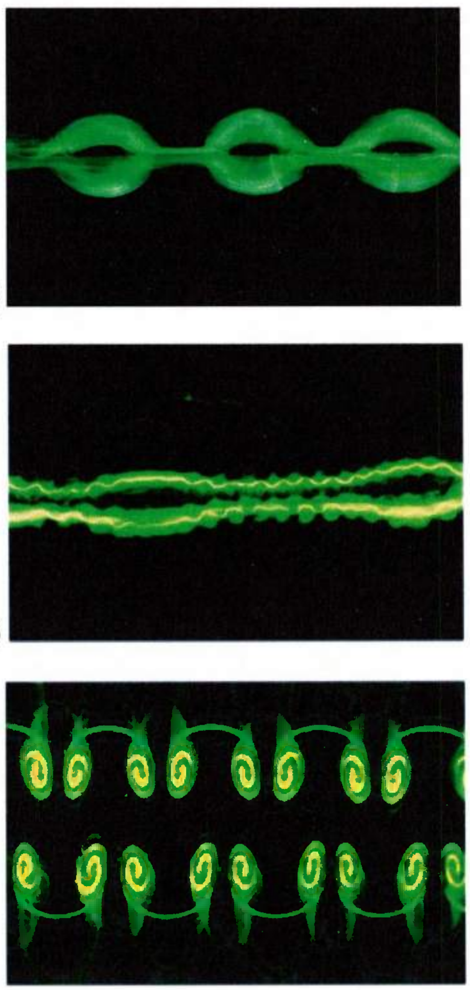


FIGURE 2: OBLIQUE VORTEX GENERATOR FACILITY Counter-rotating vortex pairs are generated near the surface of a tank of water by impulsively closing a pair of flaps (orange plates visible near the top of the diagram). These vortices travel downwards, and impinge upon a horizontal plate, or upon a plate angled down as shown above (with the green sides), causing oblique vortex pair interaction with a ground plane. The rotating plates are computer-controlled, along with the linear mechanism to carefully lower the plates into the water tank.

3. Research on Vortex-Wall Interactions

Central to this research program is the fundamental problem of the dynamics and instabilities of streamwise vortices (mostly vortex pairs) interacting with surfaces. Applications of such flows include tip vortices and junction vortices interacting with hull surfaces, their interaction with boundary layers, as well as design of vortex generators to modify surface pressures. In our first year, we reported the discovery of some new and fundamental aspects of the temporal development of vortex pairs with a surface (e.g. Harris & Williamson 2012). Several of these 3D interactions are new, and have not been studied previously in the literature. In the second year, we have further synthesized the results pertaining to long-wave instability in wall effect, and characterized such flows with more quantitative measurements of velocity and vorticity in the flow using PIV. Aside from the element of discovery, a strong motivation in our present work is to determine *generic phenomena* that will have relevance to a whole set of fluid flows, including the practical cases mentioned above as well as general turbulent flows where longitudinal vortices are present. Subsequent work will complete these studies described above and compare the new *temporally-evolving* results with *spatially-evolving* vortex dynamics, amongst other objectives.

For all wall-effect studies, a principal feature of the interaction between a vortex pair and a wall is the generation of a secondary vortex between each primary vortex and the wall. Due to an adverse pressure gradient, such vorticity can separate and roll up into a secondary vortex, which is advected around the stronger primary vortex. We discovered a short-wave instability of such secondary vortices, which was discussed at length last year (Harris & Williamson, 2012). In the present research, we have completed research concerning long-wave instability as it is influenced by wall proximity. We find that there are three regimes of vortex dynamics, depending on the height between the vortex generator and the horizontal wall. For small initial heights, the resulting topology is radically different from the classical "Crow instability," which comprises a series of large vortex rings. The net result of the interactions is a system of vertically-oriented vortex rings, due to the 3D dynamics of the secondary vortices interacting with each other. In Regime 2, at moderate initial heights of the approaching vortex pair, the increased sinuous instability amplitude pushes the secondary vortices further apart, so that they interact more with the primary vortices instead, leading to strong axial flows and the break-up of the primaries into concentrated pairs of small vortex rings which rise away from the wall region. In Regime 3, the larger initial height allows the Crow instability and the formation of the well-known periodic vortex rings to form before wall interaction. Even in this case, the wall interaction is surprising if one considers prior studies of axisymmetric rings approaching a wall. In fact, the rings that form in this flow are not necessarily axisymmetric, and one finds again the development of strong axial flows (in this case across the flow, rather than parallel to the initial vortex pair axis) and the formation of a set of small (horizontal) vortex rings, two per instability wavelength. The progress in understanding these complex flows has led to the question: *to what extent are the physical mechanisms, which lead to strong axial flow, and to periodic arrays of small vortex rings, generic to a whole class of flows where perturbed vortices are subject to locally increased circulation decay?*



(a)

(b)

(c)

FIGURE 3. Comparisons of the principal instabilities associated with a counter-rotating vortex pair, as visualized in experiment. (a) Long-wave Crow instability (see for example Leweke & Williamson, 2012). (b) Short-wave cooperative elliptic instability (Leweke & Williamson 1998). (c) Instability of the secondary vortices generated in ground effect; only the secondary vortices are shown here, because the primary vortices are purposely invisible, as they are not marked with dye (Harris & Williamson 2012). (Images not to same scale.)

As mentioned above, a fundamental physical mechanism in all modes found here comprises strong axial flows away from the point of closest approach to the wall; this causes the break-up of vortex pairs and formation of periodic arrays of small vortex rings. In fact, it is significant that, in the present work, the formation of vortex rings is a feature of all of the mechanisms in which vorticity changes topology, or is redistributed due to fundamental vortex-surface interactions.

Much of our research, during the period of support, has been concerned with longitudinal vortices in proximity to a surface. As mentioned earlier, this represents a fundamental problem with several applications. For example, streamwise or trailing vortices are often formed around vehicle surfaces, including around submersible vehicles. Aerodynamic applications, such as the trailing vortices left behind an aircraft over a runway or aircraft carrier deck, are also numerous. Vortices adjacent to a surface are also relevant to the physics of vortex generators, which seek to delay flow separation over a surface. Longitudinal vortices are also generated near the tips of control surfaces, as may be found on submarines or semi-submersibles (see again Figure 1).

There have recently been a number of studies into the interaction of streamwise vortices with turbulent boundary layers, usually formed by vortex generators. Of interest in such studies would be the formation of secondary vorticity from the surface, the downstream vortex trajectories, and the decay in vortex strength or diffusion of the vortices. We would also be interested in the break-up of the longitudinal vortices and the interaction of primary and secondary vortices and the wall. Several of the vortex-surface interactions, which we would look for in the *spatially* evolving vortices, may also be studied in a *temporal* context. This is the approach so far adopted in the present research. These interactions produce vortex dynamics phenomena which are distinctly clearer than those found in the spatial case, especially when a boundary layer is present in spatially evolving flows.

Three principal instabilities of vortex pairs, either isolated or near a surface, have been found to date. A prominent feature of this flow is a long-wavelength instability (e.g. Leweke & Williamson, *Physics of Fluids*, 2011). When this "Crow" instability (Crow, 1970) grows large enough, portions of the displaced vortices can approach each other and "pinch off," or reconnect, into an array of vortex rings, as seen in Figure 3a. Widnall et al. (1974) and Tsai & Widnall (1976) proposed a mechanism for the short-wave "Widnall" instability in flows with strained concentrated vortices, of which the counter-rotating vortex pair is one example. It involves complex perturbations leading to *internal deformations* of the vortex cores, as may be seen in Figure 3(b), taken from laboratory experiments (Leweke & Williamson, 1998).

The above brief introduction pertains to the dynamics of counter-rotating vortices outside the influence of a wall surface. It is now of interest to see to what extent the presence of the wall influences these dynamics and instabilities. A principal research study concerns the short-wave instability of the secondary vortices generated by primary vortex-wall interaction (found in Harris & Williamson, 2012), and seen in Figure 3c. Much of the research during this period of support has concerned the influence of a wall on the evolution of the *long-wave instability* and dynamics.

We briefly summarize some essential characteristics of the influence of ground proximity on the development of long-wave instability which have come to light in this research. This problem has not been previously observed in the literature to our knowledge and, as such, the phenomena we observe are new, including the periodic axial flows and formation of rebounding systems of vortex rings, both of which are ubiquitous to these vortex-surface interactions We shall not discuss here the initial instability itself. It is clear that if the initial height of the vortex pair above the surface (h_0) is large compared with the inter-vortex spacing (b_0), then the Crow instability, and the eventual redistribution of vorticity into vortex rings, will occur prior to wall interaction. Correspondingly, one must also expect that if the vortex pair is generated below a critical height to the surface (h_0/b_0 below a critical value), then there will not be enough time for the long-wave instability to take hold before the vortices are separated from each other in wall effect; one might expect the long-wavelength instability will be inhibited. It is significant to evaluate such a suggested *critical height*, and the parameters upon which it depends.

In this Final Report, we take three cases, or three initial heights, which will exhibit three different regimes. For each regime, it is useful to observe the stage that the long-wavelength Crow instability would have reached at the level of the wall, if there had been no wall surface. This is shown in Figure 4 below.

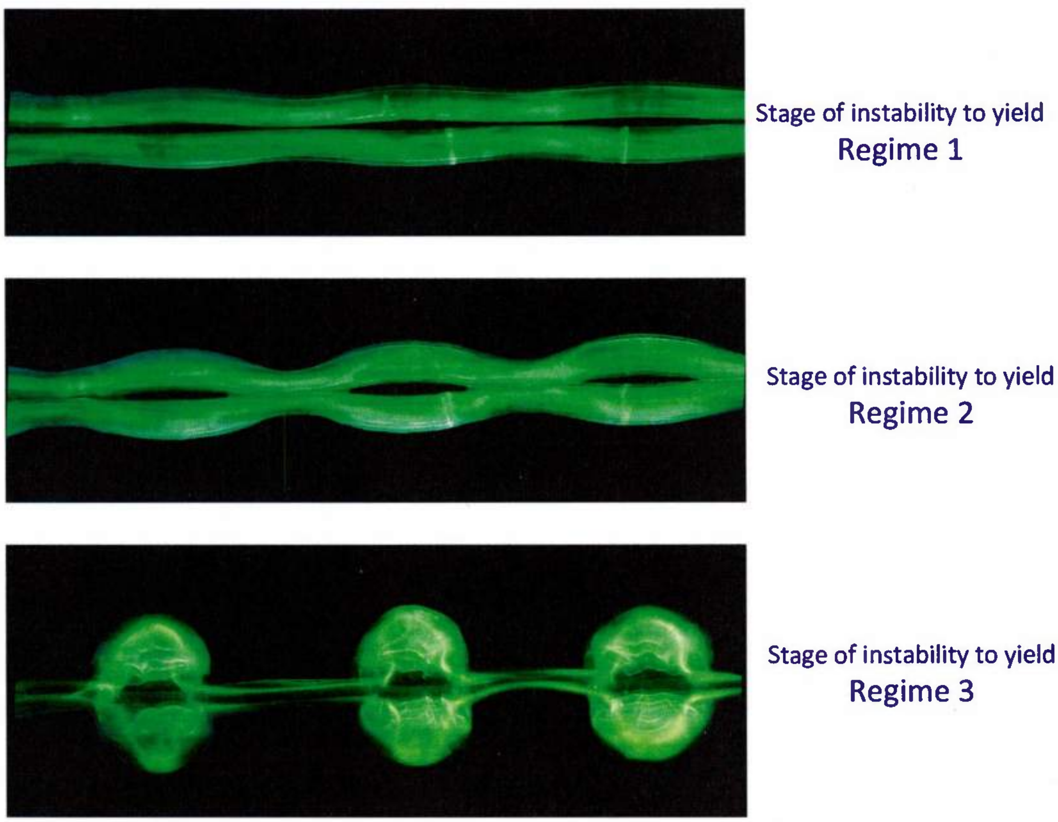


FIGURE 4. Stages in the Long-wave Crow instability which interact with the wall. For each regime, it is useful to observe the stage that the long-wavelength instability would have reached at the level of the wall, if there had been no wall surface. (For a straightforward presentation of the long-wave instability itself in experiment, see Leweke & Williamson, 2012).

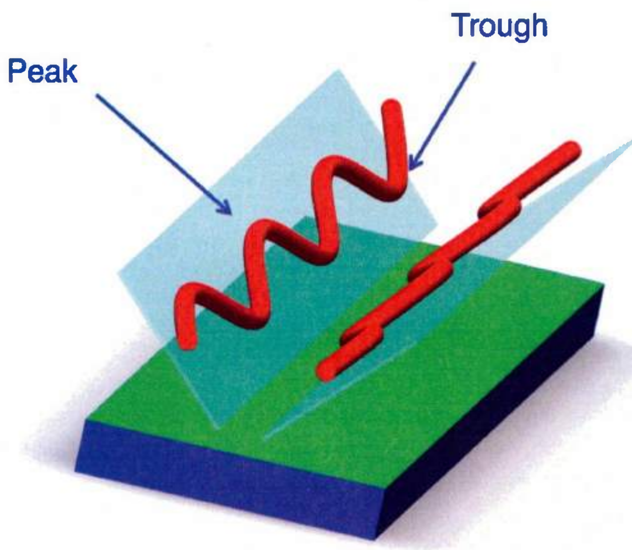


FIGURE 5. Schematic showing the approach of a pair of counter-rotating vortices to a solid boundary. The waviness resides within planes at close to 45° to the horizontal. Note our definition of the "peak" and "trough" along the span.

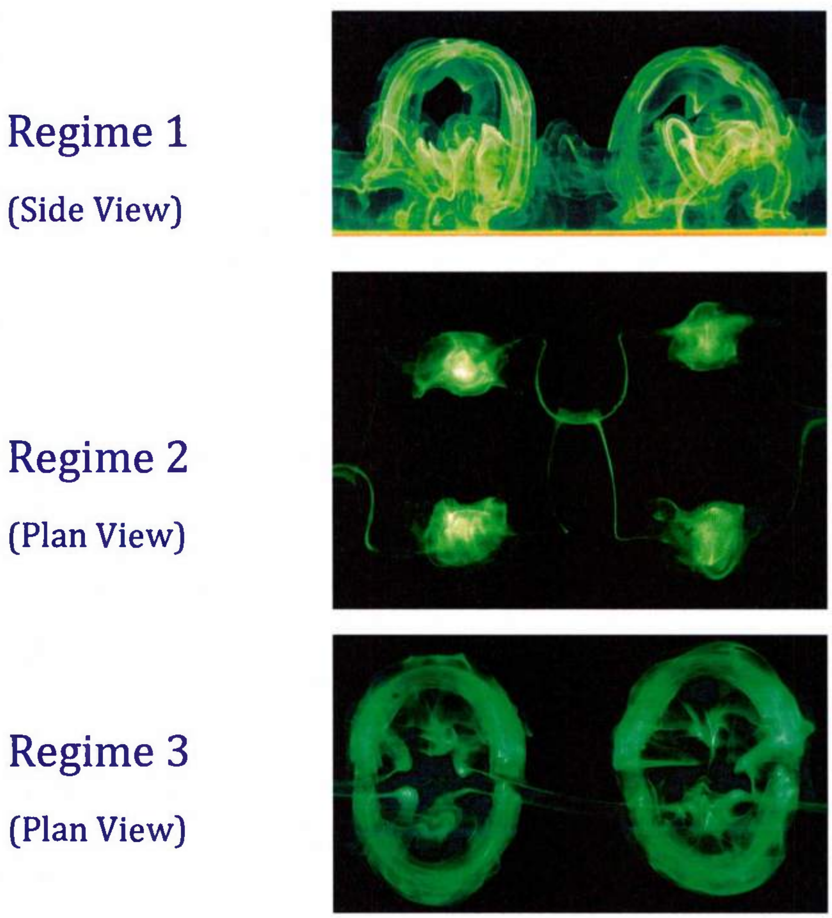


FIGURE 6. Images of the principal vortex dynamics for the three modes of vortex-wall interaction found here. Regime 1 exhibits vertical vortex rings, actually created by secondary-secondary vortex interactions ($h_0/b_0 = 5.0$). Regime 2 (for moderate initial heights, $h_0/b_0 = 7.5$), shows the final configuration of two vortex rings (per instability wavelength) rising away from the wall after the rest of the vortices have been evacuated by the vorticity transport away from the troughs (due in turn to the strong axial pressure gradients). Regime 3 shows the interaction of vortex rings with the wall surface at these larger initial heights ($h_0/b_0 = 10.0$).

We include a schematic in Figure 5, which shows the long-wave instability close to the ground, and indicates that the waviness of this displacement mode resides in planes that are oriented at approximately 45° to the horizontal. Therefore the "trough" of each wavelength will reach and interact with the wall surface first, before the peak of each wavelength. This is key to the resulting vortex dynamics and is discussed below.

In essence, we find three regimes of vortex-surface interaction, which are represented by the images in Figure 6, and which occur in ranges of the initial vortex heights, h_0/b_0 :

Regime I: Vertical vortex rings (For small heights above the wall; $h_0/b_0 \sim 5$ in this example).

For small initial heights of the vortex pair above the ground, there is only little time for the long-wavelength instability to develop. The waviness might be small, but even small effects can be strongly amplified in the presence of the wall. The part of the vortex which first interacts with the wall, namely the “trough” (see figure 5), reduces its strength rapidly as secondary vorticity is generated on the wall, and there is significant diffusion and cancellation of vorticity. The net effect is a strong axial pressure gradient pushing fluid and vorticity away from the trough towards the peak. The vertical vortex rings, evident in Figure 6a, are actually formed by interaction between the secondary vortex loops on each side of the flow, and are concentrated at the "peaks" of the primary vortices. Further discussion of Regime 1 is included below.

Regime II: Double vortex rings. (For moderate initial heights $h_0/b_0 \sim 7.5$).

In this case, the secondary vorticity (at the "peak") does not have the opportunity to interact with secondary vorticity from the other side of the flow, because the amplitude of the instability causes the "peaks" to move apart. A strong axial flow is produced by the weakened trough circulation which has a higher pressure than the stronger peak circulation. The strong axial flow has the effect of stripping away the vorticity from the trough region and concentrating the vorticity at the peaks. In this case, we find two regions of concentrated vorticity per wavelength, as seen in Figure 6b. Further measurements indicate that these vorticity structures develop into small vortex rings, which rise vertically upwards away from the wall (towards the reader in Figure 6b).

Regime III: Ring-Wall interactions. (For larger initial heights $h_0/b_0 \sim 10$).

For heights above a critical value (where $(h_0/b_0)_{CRITICAL} \sim 8.5$), the instability evolves into classical "Crow-type" vortex rings before contact with the ground, and we see in this case the enlarging of the ring diameter as it comes under the influence of the wall boundary condition (See Figure 6c). It may be supposed that this mode involves axisymmetric vortex rings interacting with the wall, as studied in previous literature, where the overall ring diameter spreads out strikingly. However, this is not necessarily the case; the rings here may *not* be axisymmetric, since the central portions of each ring dip down towards the wall and interact with the wall first (see Leweke & Williamson, 1998). As a result, it is fascinating that the strong axial flows now occur transverse to the original vortex pair configuration, rather than parallel to the original pair, as in the case of Regimes 1 and 2. The net result is the formation of pair of small vortex rings within each instability, although the final state is unclear.

We now focus on Regime 1, which leads to the principal vortex dynamics in the form of the vertical rings in Figure 6a. A key to the appearance of the vertical rings is the interaction between the secondary vortices. Firstly, looking at Figure 7, showing only the secondary vortices being visualized by our technique, we see that a tongue of secondary vorticity gets wrapped around the (invisible) primary first, because the "trough" approaches the ground first. An interpretation of this tongue is made in the schematic of Figure 8. The presence of the secondaries produces an axial flow from the trough towards the peak, in each wavelength of the flow. This axial flow (which is shown in both Figures 7 and 8) causes both the primary and secondary vorticity to concentrate at the "peaks" of the wavy vortices. The axial flow is a consequence of the fact that the circulation of the vortex at the trough is sharply reduced by vorticity cancellation of the primary vortex in the close proximity with the secondary vorticity produced at the wall. This reduction in circulation causes an increase in pressure compared to the relatively stronger vortex and higher circulation found at the peak, where the pressure is lower. To illustrate this analytically, the following expression gives the pressure difference from ambient at the center of a Rankine vortex

$$p - p_0 = -\frac{\rho \Gamma^2}{4\pi^2 a^2}$$

where p is the pressure, p_0 is the ambient pressure, ρ is fluid density, Γ is circulation, and a is the vortex core size. Similar but more complex calculations can be performed for a Lamb-Oseen vortex, which has a Gaussian vorticity distribution. In both cases, reducing the circulation at a particular point in a vortex causes the pressure to rise there, leading to the axial flow that we observe.

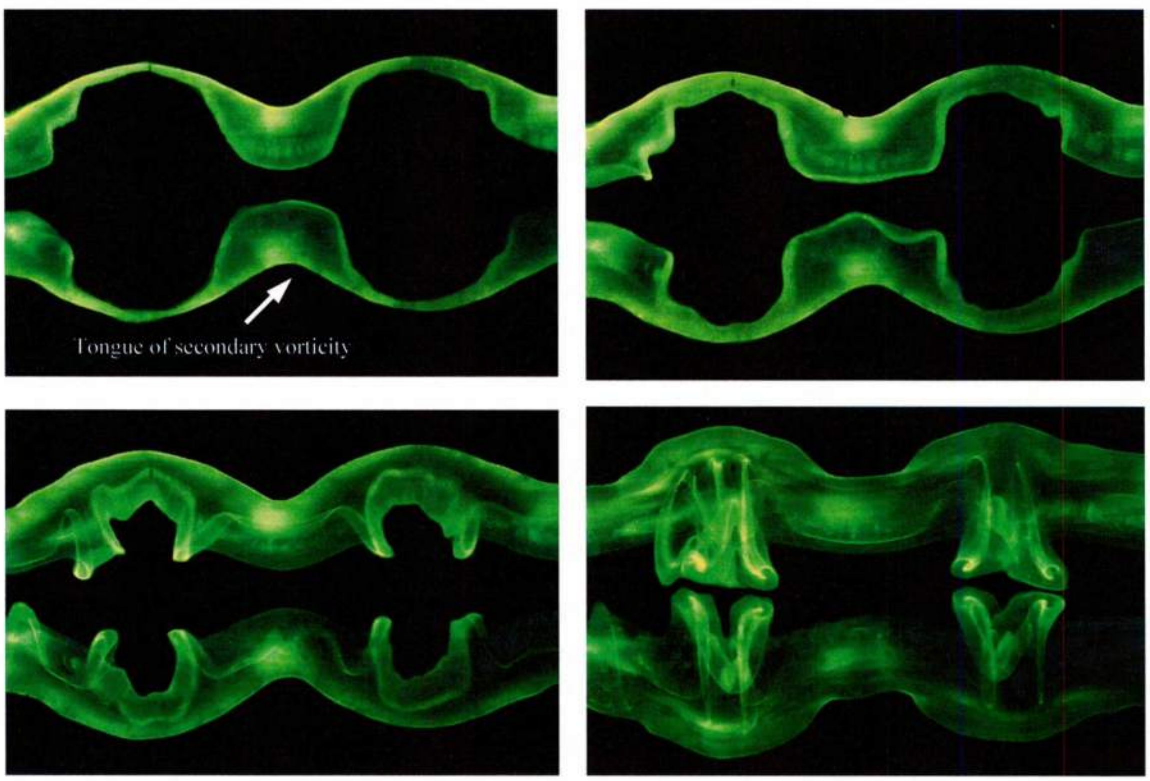


FIGURE 7. The secondary vortices are visualized for Regime 1, while keeping the primaries invisible (although the primaries wrap secondary vorticity around themselves and so can be seen). The tongue of vorticity at the trough gets wrapped around the primaries, and there is much cancellation of vorticity here. The lower circulation is associated with the higher pressure, and leads to an axial pressure gradient and to a flow from trough to peak. The mutual interactions between vortex loops in the secondary vortices (near to the peaks of the primaries) lead to development of the vertical vortex rings.

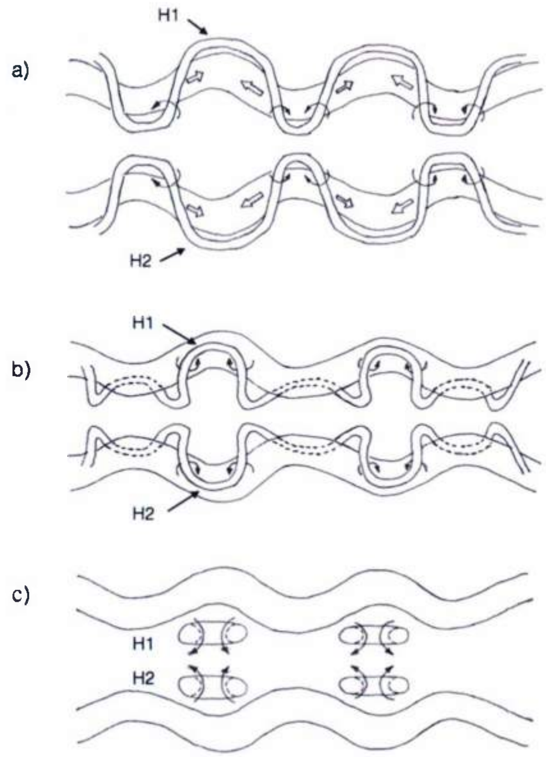


FIGURE 8. Plan View Schematic of the secondary vorticity, which is principally responsible for the axial flow from trough to peak, and it evolves into the vertical vortex rings, as seen above. H1 and H2 are the heads of the secondary vortex loops, which pinch off to form loops.

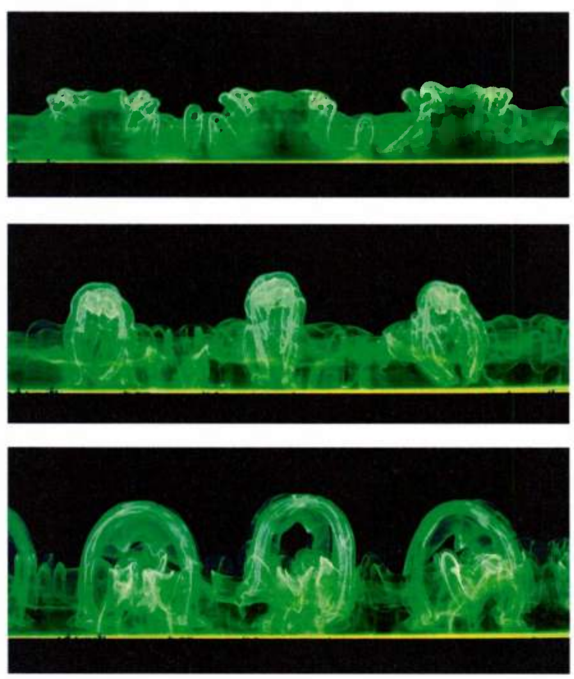


FIGURE 9. Side View. This is a view of the secondary vortices for Regime 1, seen from the side and showing the development of the vertical vortex loops that are characteristic of low initial vortex generation heights. The sense of rotation of the secondary vorticity generated at the peak cross section is such that it rotates up away from the wall and moves toward the centerline dividing the two primary vortices. There, it interacts with the neighboring secondary vortex and forms the expanding vertical vortex rings seen above.

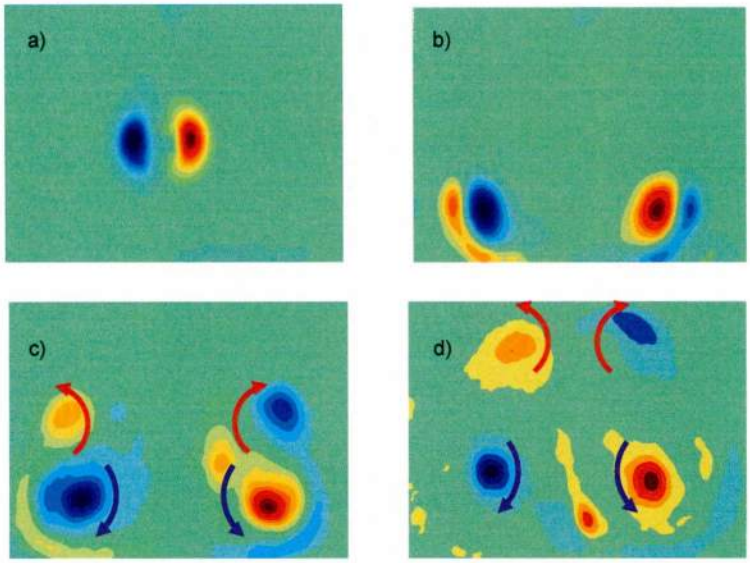


FIGURE 10. Contours of vorticity in a plane transverse to the axes of the primary vortices at the peak cross section are shown. In (a), the primary vortex pair descends toward the wall. The generation of secondary vorticity there (b) eventually leads to the separation of a discrete secondary vortex (c). This secondary vortex is advected around the primary vortex. Because the vortices are in relatively close proximity at this low initial height, the two secondary vortices interact quite strongly with each other and form the vertical vortex rings described above. Here, one can see these rings in cross section as they move up and away from the wall, indicated by the red arrows (d).

The resulting axial pressure gradient leads to a flow from the trough to the peak, which results in the concentrated vorticity at the peak. In Regime 1 this mechanism is not as strong as for Regime 2, where the vortices become almost completely stripped in the trough region. The secondary vortices form loops near the primary vortex peaks, and the interaction of the secondary vortex loops on each side of the flow then results in the set of vertical vortex rings, seen earlier. The development of these loops, growing vertically out of these interactions, is seen in the sequence of images in Figure 9.

In Figure 10, we show a sequence of vorticity plots in the cross-section of the primary and secondary vortices. These indicate the interesting phenomenon whereby the secondaries (with the red arrows) are advected around the primaries (blue arrows), but as they reach the top of the primaries they then interact with each other and rise up together above the primary vortices. These vorticity images are taken at the peak cross section, and therefore the secondary vortices in Figure 10(d) represent a cut through the "vertical vortex rings" discussed earlier. It is interesting that the secondaries have a choice whether to meet up above the primaries and then induce each other upwards as a pair (the case we have been discussing for Regime 1), or whether the primaries are so far apart that the secondaries advect further around the primaries and travel downwards between the primaries; this would be the case for vortices not subject to the Crow instability.

The interaction of vortices is quite different for Regime 2, as shown in Figure 11. In Regime 1, we observed that there is an interaction between the secondary vortices (on each side of the flow) at the peak, and the secondaries induce each other upwards away from the primaries. In Regime 2, the amplitude of long wave instability has grown further, and this inhibits the secondary-secondary interaction (because of the fact that they are further apart), and instead there is a stronger secondary-primary interaction.

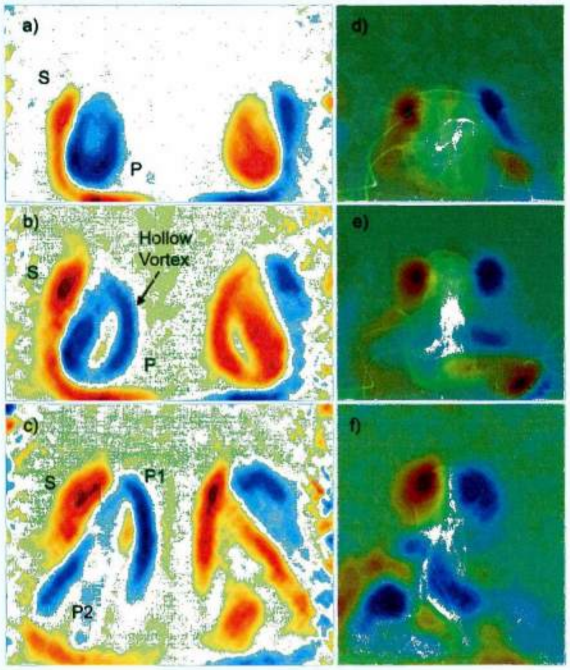


FIGURE 11. Contours of vorticity in a plane transverse to the axes of the primary vortices at the peak cross section are shown. The images (a) to (c) show the pinching off of primary vorticity (P) into P1 and P2, and from there we have a pairing between vorticity (P1) and secondary vorticity (S), into vortex rings rising vertically away from the wall. Images (d) to (f) exhibit an orthogonal light sheet, with simultaneous vorticity superposed on the dye structure, making clear that these are indeed vortex rings.

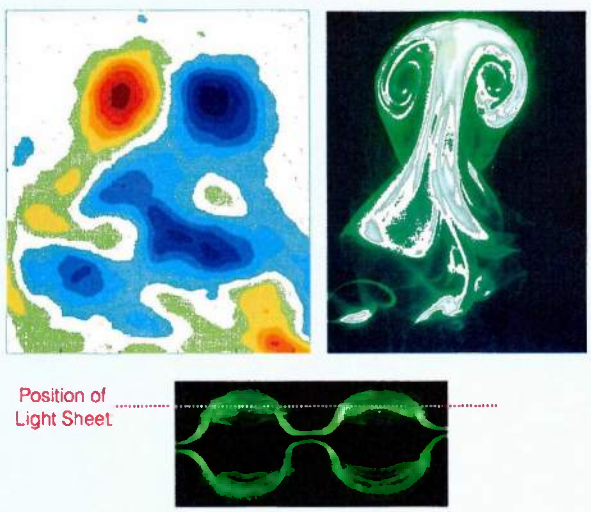


FIGURE 12. With a light sheet positioned as shown in (c), the view normal to this plane shows from vorticity measurements (a) and from visualization (b), both very clearly exhibit the vortex rings rising away from the wall

In Figure 11, the axial flows cause a "hollow" primary vortex to form at the peak location (vortex P). Subsequently, the secondary (vortex S) forms a pair with part of the primary (vortex P1), and they induce each other upwards away from the horizontal surface. Of course these vortex dynamics are happening in three dimensions, and what actually evolves, due to vortex reconnection is the formation of a vortex ring structure, instead of 2D vortex pairing. Such vortex rings in cross section are shown by the PIV vorticity measurements and visualizations in Figures 11 and 12.

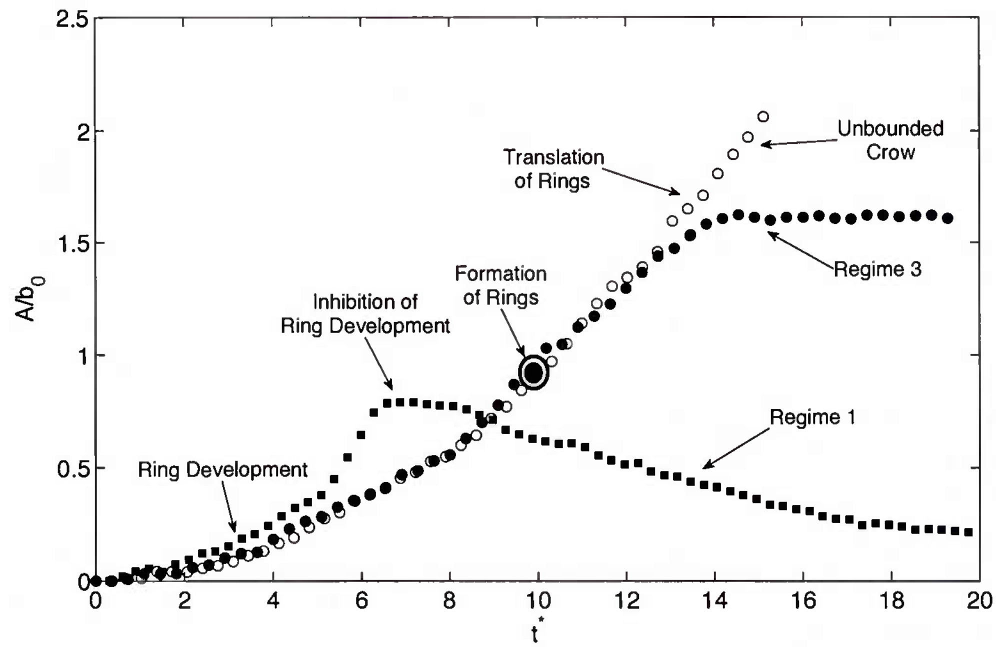


Figure 13: A fundamental feature of wall effect: In Regime 1, the ground serves to inhibit the growth of instability amplitude as shown by the solid square symbols. The open symbols show the growth of amplitude for the Crow instability out of ground effect. Regime 3 is represented by the solid circles, and follow the curve of the isolated vortex ring development, until wall effect is felt for t*>14.

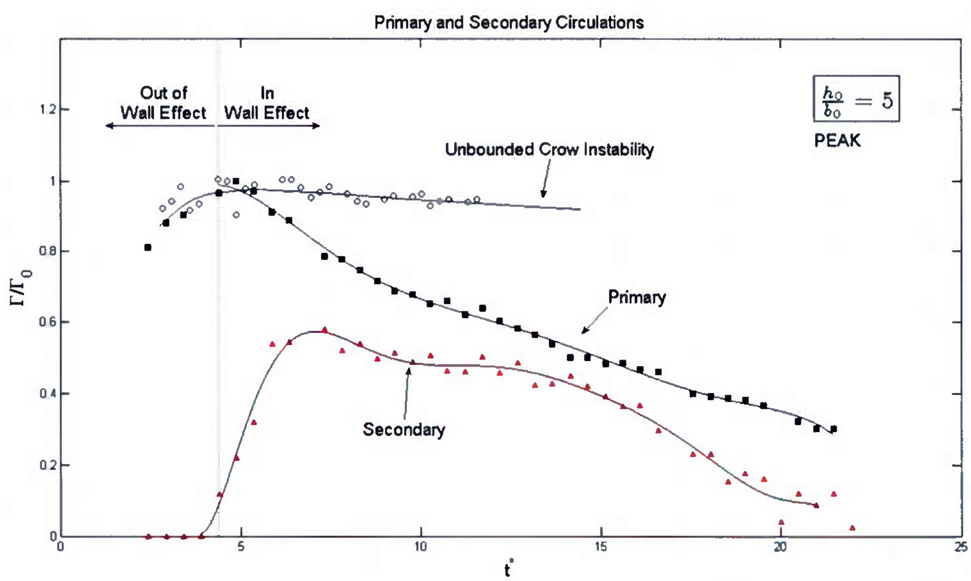


Figure 14: A second fundamental feature of wall effect: In all regimes of vortex-ground interaction, a dramatic reduction in primary vortex strength (solid symbols) is found, due to the interaction with the forming secondary vortex. This contrasts with the almost steady vortex strength of the primary vortices that form into vortex rings, out of ground effect.

Finally, we show briefly two fundamental features of wall effect. For initial heights corresponding to Regime 1 in Figure 13, the development of the long wavelength instability is clearly inhibited by the presence of the wall. Once the secondary vorticity starts to grow at t*=5, and the primary circulation diminishes, the amplitude of the instability actually deceases slowly, in stark contrast with the unbounded Crow instability case, whose amplitude grows rapidly. In Figure 14, the growth of secondary vorticity corresponds with the reduction in primary circulation, which is quite different from the circulation in the Crow instability vortices in the unbounded case, whose circulation is diminishing very slowly. The effect on the primary vortex circulation decay by the wall is quite significant.

4. Concluding Remarks

4.1 Characteristics of Vortex-Wall interactions

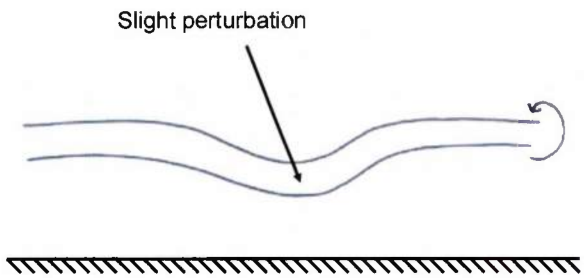
In summary, the principal characteristics of the long-wave instability in the presence of the wall are as follows:

- Even a very slight waviness in each vortex, as it approaches the wall, can trigger a large pressure gradient and axial flow that strips away the vortex filaments at the trough and forms concentrated 3D vortices at the peak. (See Figure 15 for a sequence of events leading to these axial flows and concentrated vortices.) One might conclude that the effect of a surface interacting with a small perturbation on an otherwise parallel vortex is enough to cause surprisingly large three-dimensional effects. This might be seen as a fundamental characteristic for a vortex aligned with a surface.

- The concentrated vortices at the peak either evolve into vertical vortex rings of large diameter (coming from the secondary-secondary vortex interactions) or evolve into smaller horizontal vortex rings (coming from the primary-secondary vortex interactions), which rise up away from the wall. In essence, vortex rings are ubiquitous in these flows, despite the apparent complexity of these vortex interactions. They generally seem to appear out of the remnants of the primary vortex "impact" with a wall.

- In general, the Crow instability is inhibited by the presence of the ground, if the initial vortex pair height is below a critical value (where $(h_0/b_0)_{CRITICAL} \sim 8.5$). It is particularly important to note that the precise vorticity structures produced are highly dependent on the extent to which the Crow instability has developed prior to contact with the boundary. In other words, this phenomenon is sensitive to the initial height above the surface.

- It is relevant to mention that the presence of a wall is not the only mechanism that can cause strong axial flows along a vortex. The long-wave Crow instability, isolated from a wall, has a strong reduction in circulation strength as the vortices in the pair come close to each other at the trough, and in this case also there is a strong axial flow, as a part of the "pinch-off" process to form the classical Crow instability vortex rings.



Slight perturbation

↓

Accelerated Circulation Decay

↓

Axial Pressure Gradient

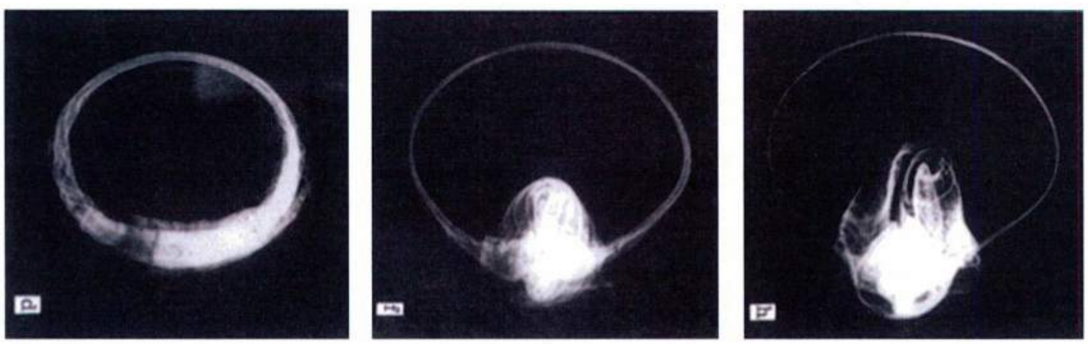
↓

Strong Axial Flows

↓

Formation of Concentrated Vortices

Figure 15. The scenario of effects which result in strong axial pressure gradients and strong vortex core flows, ultimately causing marked topology changes of vorticity in a flow. This sequence of events seems to be generic to a whole set of flows, where locally there is vorticity cancellation at some point along a span of a vortex, whether such cancellation is caused by vortex-wall or vortex-vortex interactions.



T.T. Lim (1989)

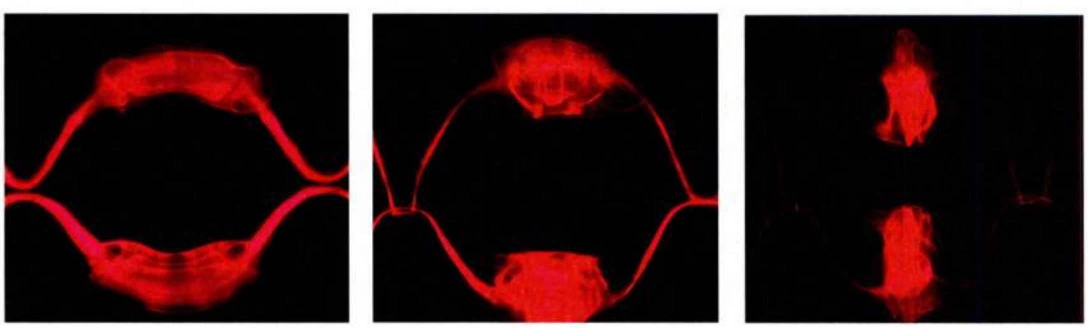


Figure 16. Remarkably similar phenomena are found between our flow (lower images) and the flow due to an oblique approach of a vortex ring to a wall (upper sequence of images, TT.Lim 1989).

4.2 A generic mechanism to cause axial flows and to form concentrated 3D vortices

There are two significant features of the flow when a perturbed longitudinal vortex is in close proximity to a wall; firstly, the presence of a strong axial flow; and secondly, the formation of small vortex rings, as a part of the three-dimensional version of the "vortex rebound" phenomenon. These features are listed as follows:

- Axial vortex core flows: The first phenomenon is the significant axial flow in each vortex tube. Because the Crow instability causes wavy displacements in a plane oriented at approximately 45 degrees to the horizontal, the "troughs" of the sinusoidal vortex tube, at which the vortices are closest together, interact with the ground before the "peaks." The result is a strong motion of fluid away from the region of closest approach (the troughs), producing bulbous regions of vorticity at the peaks of the vortex (see Figure 6b).

- Rebounding vortex rings. The second phenomenon occurs as a result of the interaction of primary-secondary vorticity. A system of small vortex rings is formed out of these interactions. This appears to be one of the phenomena that represent the three-dimensional version of "vortex rebound." In the two-dimensional version, the presence of 2D secondary vortices causes the primary vortex pair to "rebound" away from the wall, after impinging on the wall. In a sense, the primary and secondary vortices pair up to form their own vortex pairs. The three-dimensional version, when a wavy primary vortex approaches a boundary, sees the formation of vortex rings out of primary and secondary vorticity. So, in one case we have vortex pairs, and in the other case, we have vortex rings, which result from primary-secondary vortex interactions. In both cases, they represent an apparent "vortex rebound."

The strong axial flow inside the vortices whenever a vortex tube is locally disturbed towards a wall would appear to be a basic generic phenomenon, which we have characterized and measured and is more-fully described in (Asselin & Williamson, Submitted to *Journal of Fluid Mechanics*, 2015). In fact, the scenario in Figure 15 shows that an accelerated circulation decay comes about from locally strong primary-secondary interactions. Similar changes in the flow topology can occur whenever there is locally a rapid circulation decay between two vortices pushed together, such as for the pinch-off stages of the Crow instability when the vortex pair changes into a set of vortex rings. These effects are seen when a flow becomes three-dimensional either from vortex-wall interaction or from vortex-vortex interactions.

The remarkable similarities between the flow coming from the oblique approach of a vortex ring to a wall (Lim 1989) and the flow we have here are shown in Figure 16. This suggests very strongly that similar vortex dynamics are occurring in the two flows. T.T. Lim's vortex ring is subject to strong axial flows away from the point of the ring which impacts the wall first, towards the part of the ring further from the wall, which is equivalent to our trough-to-peak flow in the present case. These effects, leading to axial pressure gradients and axial vortex core flows, and changes in topology typically involving the appearance of new vortex rings, would appear to be *generic features* of some flows where there is a localized accelerated circulation decay in a vortex..

We are pushing forward to publish all of these new results (including recent submission to *Journal of Fluid Mechanics)*, and to further study these fundamental phenomena. Studies involving the oblique approach of vortices to a wall, are an obvious extension of the present work. It is clearly of interest to study the evolution of longitudinal vortices and the development of 3D instabilities in the spatial flow to compare with the present temporal studies. The development of turbulent diffusion and decay of the principal vortices, and their effect on the pressure field and shear stresses on the surface, are of fundamental and practical interest.

4.3 Perturb the wall rather than the vortices: formation of rebounding vortex ring arrays.

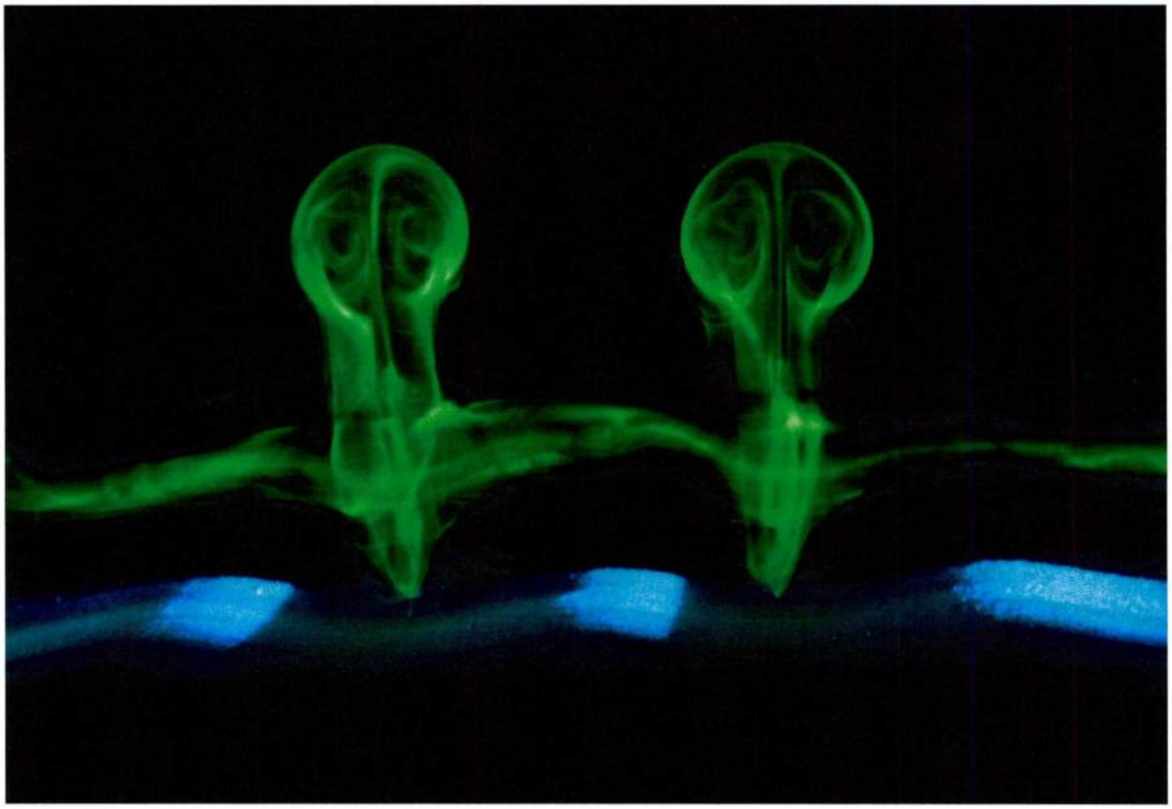


FIGURE 17. We have been exploring the concept of vortex-surface interaction where, instead of having a wavy vortex impinge upon a flat wall, we have a straight vortex interacting with a wavy wall. This is one of our first images of the resulting phenomenon, clearly showing an array of small vortex rings rising away from the wall, after the vortex-surface interactions. All of these vortex-surface interactions are quite surprising, but there are strong similarities with the wavy vortex case.

A fascinating very recent result is represented by the image in Figure 17, where we have 2D vortex pairs impinging on a perturbed surface, as distinct from wavy vortices impinging on a flat surface. Quite surprisingly, the principal vortex dynamics are remarkably similar but clearer than what happens on a flat surface in Regime 2 of the previous figures. The net result of the vortex-wall interactions are to produce an array of vortex rings which translate upwards and away from the horizontal wavy surface. The peak of the wavy surface is equivalent to the trough of the wavy vortices in the previous studies reported here. The surface peak comes into contact with the vortices sooner that the surface trough, leading to axial flows from peak - to - trough of the surface waviness. The clear image of these vortex rings in this instance only show such vortex rings on the near side of the flow, while in reality there are two other vortex rings on the further side of the flow field (not colored by dye), essentially producing two rising vortex rings per wavelength of the wavy ground plane.

5. Publications from research fully or partially supported by ONR.

T. Leweke, S. LeDizes & C. H. K. Williamson (2016). To appear. "Dynamics and Instabilities of Vortex Pairs", Invited Review for *Annual Review of Fluid Mechanics*. In press.

D.J. Asselin & C. H. K. Williamson (2015) Evolution of vortex pairs subject to the Crow instability in wall effect. *Bull. American Physical Society*, **60,** H19.00003.

D.J. Asselin & C. H. K. Williamson (2015) Vortex Pair Impinging on a Horizontal Ground Plane. *HELIX 2015: Fluid-Structure Interactions and Vortex Dynamics in Aerodynamics*: Porquerolles, France.

C.H.K. Williamson, T. Leweke, D.J. Asselin & D.M. Harris (2014) Phenomena, dynamics and instabilities of vortex pairs. *Fluid Dynamics Research*, **46**, 061425 (23pp).

D.J. Asselin & C. H. K. Williamson (2014) Evolution of vortex pairs subject to the Crow instability in wall effect. *Bull. American Physical Society*, **59**, H18.00008.

P. Luzzatto-Fegiz & C. H. K. Williamson (2013) The structure and stability of uniform and distributed opposite-signed vortex pairs. Submitted to *Journal of Fluid Mechanics*.

D.J. Asselin & C. H. K. Williamson (2013) "Vortex Pair impinging on a horizontal ground plane". *Physics of Fluids*, **25**, 091104.

P. Luzzatto-Fegiz & C. H. K. Williamson (2013) Book Chapter: Investigating stability and finding new solutions in conservative fluid flows through bifurcation approaches. *Nonlinear Physical Systems* (Ed. O.N. Kirilov; D.E. Pelinovsky). Wiley & Sons, Inc., Hoboken, USA. Chapter 10. (In print 27 Nov 2013; Online 10 Feb 2014)

P. Luzzatto-Fegiz & C. H. K. Williamson (2013) Computing steady vortex flows of prescribed topology. *Topological Fluid Dynamics II* (Eds. H.K.Moffatt & K.Bajer), Procedia *IUTAM*, 67-76.

P. Luzzatto-Fegiz & C. H. K. Williamson (2012) Stability of steady two-dimensional flows through "Imperfect Velocity-Impulse" diagrams. *Journal of Fluid Mechanics*, **706**, 323-350.

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P. Luzzatto-Fegiz & C. H. K. Williamson (2012) Structure and stability of the finite-area Karman vortex street. *Physics of Fluids*, **24**, 066602.

C. H. K. Williamson (2013) "Flow Phenomena, Vortex Dynamics and Instabilities of Vortex Pairs", Invited Keynote Speaker at the *IUTAM Symposium on Vortex Dynamics: Formation, Structure and Function,* Fukuoka, JAPAN. March 10-14, 2013

D.J. Asselin & C. H. K. Williamson (2013) "Formation of small-scale vortex rings from vortex pairs close to the ground". *Bull. American Physical Society*, **58**, L25.00002.